MATH 108 Fall 2019 - **Problem Set 1**

due October 4

1. Write the truth table for each propositional form, and determine if it is a tautology, a contradiction, or neither.

   (a) $P \Leftrightarrow P \wedge (P \vee Q)$.

   (b) $[Q \wedge (P \Rightarrow Q)] \Rightarrow P$.

   (c) $P \wedge (P \Leftrightarrow Q) \wedge \sim Q$.

   (d) $(P \Rightarrow Q) \Leftrightarrow (Q \Rightarrow P)$.

   Here is an example truth table that you can use as a template:

P	Q	$P \vee Q$	$P \wedge Q$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

2. Rewrite each proposition in English. You may use mathematical expressions (e.g. "$x = 0$") in your answers but replace all the logical symbols. Take the universe to be all real numbers.

   (a) $(\forall x)(\forall y)[(xy > 0) \vee (xy < 0)]$.

   (b) $(\exists x)(\forall y)(x + y = 0)$.

   (c) $(\forall y)(\exists x)(x + y = 0)$.

   (d) $(\forall x)[x > 0 \Rightarrow (\exists y)(xy = 1)]$.

   (e) $(\forall y)(\exists! x)[(x \leq y) \wedge (y \leq x)]$.

   (f) $(\forall y)(\exists! x)(y = x^2)$.

3. Determine if each proposition in Problem 2 is true or false in the universe of all real numbers. Give a short justification for each answer.

4. Let x be a real number. For each proposition, write the contrapositive. Then prove the proposition by contraposition.

   (a) If $x^2 + 2x < 0$, then $x < 0$.

   (b) If $x(x - 4) > -3$, then $x < 1$ or $x > 3$.

5. Let a and b be positive integers. Prove each proposition by contradiction.

   (a) If a divides b, then $a \leq b$.

   (b) Either a and b are odd, or ab is even.

(c) If $a < b$ and $ab < 4$, then $a = 1$.

6. For x a real number, $\lfloor x \rfloor$ denotes the "floor" of x, which is the largest integer less than or equal to x. Prove using cases that for all integers k, the value of $\lfloor k^2/2 \rfloor$ is even.

7. Let x, y and z be three real numbers in the interval $[0, 1]$. Prove that there exists a pair of two of the three numbers that are at distance $\leq 1/2$ apart. [Hint: You can assume without loss of generality that $x \leq y \leq z$. Why is it sufficient to only consider this case?]