MATH 108 Fall 2019 - **Problem Set 9**

due December 2

1. Prove that $\mathfrak{c} + \mathfrak{c} = \mathfrak{c}$ (where $\mathfrak{c} = |\mathbb{R}|$, the cardinality of the continuum).
2. Order the following cardinalities: $|(0,1)|$, $|[0,1]|$, $|\{0,1\}|$, $|\{0\}|$, $|\mathcal{P}(\mathbb{R})|$, $|\mathbb{Q}|$, $|\emptyset|$, $|\mathbb{R}^2|$, $|\mathcal{P}(\mathcal{P}(\mathbb{R}))|$, $|\mathbb{R}|$, $|\mathcal{P}(\mathbb{Q})|$.
3. Determine whether each algebraic structure is a group. If no, which properties does it fail? If yes, is it abelian? Find an identity element if one exists.
   (a) $(\mathbb{Z}_{>0}, +)$.
   (b) $(\mathbb{Z}/4\mathbb{Z}, +)$.
   (c) $(\mathbb{Z}/4\mathbb{Z} \setminus \{\overline{0}\}, \cdot)$.
   (d) (The set of functions $\{1,2,3\} \to \{1,2,3\}$, composition).
   (e) (The set of bijective functions $\{1,2,3\} \to \{1,2,3\}$, composition).
   (f) (The set of 2×2 real matrices with determininant 1, matrix multiplication).
4. Write the Cayley table for the following finite algebraic structures.
   (a) $(\mathbb{Z}/4\mathbb{Z}, +)$.
   (b) $(\mathbb{Z}/4\mathbb{Z}, \cdot)$.
   (c) (The set of bijective functions $\{1,2,3\} \to \{1,2,3\}$, composition).