MATH 108 Winter 2019 - **Problem Set 2**

due January 25

1. (a) Prove that there exist integers m and n such that $3m + 4n = 1$.
   (b) Prove that there does not exist integers m and n such that $3m + 6n = 1$.

2. For all integers x, prove that x is divisible by 6 if and only if x is divisible by both 2 and 3.

3. Let $A = \{1, 2\}$ and $B = \{1, 4, 5\}$.
   (a) Find $A \cup B$.
   (b) Find $A \times B$.
   (c) Find $\mathcal{P}(A)$.

4. Let A, B, C, D be sets. For each proposition, give a proof or a counterexample.
   (a) $(A \cup B) \cap C \subseteq A \cup (B \cap C)$.
   (b) If $A \cap C \subseteq B \cap C$, then $A \subseteq B$.
   (c) $\mathcal{P}(A) \setminus \mathcal{P}(B) \subseteq \mathcal{P}(A \setminus B)$.
   (d) If A and B are disjoint, then $A \cap C$ and $B \cap C$ are disjoint.
   (e) If $C \subseteq A$ and $D \subseteq B$ then $D \setminus A \subseteq B \setminus C$.
   (f) If $A \cap B \cap C = \emptyset$, then A, B, C are pair-wise disjoint.

5. Let A be the set of positive integers that are not perfect squares. Let P be the set of prime numbers. Prove that $P \subseteq A$.

6. (a) Prove that if U and V are finite sets then
   $$|U| + |V| = |U \cup V| + |U \cap V|.$$
   (b) Prove that if U and V are finite sets then $U \cup V$ is finite.

7. Prove by example that there exist sets A and B with $A \subsetneq B$ and a function $f : B \to A$ that is injective (1-to-1).